Security of Continuous Variable Quantum Cryptography

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Abstract

We discuss a quantum key distribution scheme in which small phase and amplitude modulations of CW light beams carry the key information. The presence of EPR type correlations provides the quantum protection. We identify universal constraints on the level of shared information between the intended receiver (Bob) and any eavesdropper (Eve) and use this to make a general evaluation of security. We identify teleportation as an optimum eavesdropping technique.

I. INTRODUCTION

The distribution of random number keys for cryptographic purposes can be made secure by using the fundamental properties of quantum mechanics to ensure that any interception of the key information can be detected [1–3]. In particular the act of measurement in quantum mechanics inevitably disturbs the system. Further more, for single quanta such as a photon, simultaneous measurements of non-commuting variables are forbidden. By randomly encoding the information between non-commuting observables of a stream of single photons any eavesdropper (Eve) is forced to guess which observable to measure for each photon. On average, half the time Eve will guess wrong, revealing herself through the back action of the measurement to the sender (Alice) and receiver (Bob). There are a number of disadvantages in working with single photons, particularly the strong restrictions on data rates. Also it is of fundamental interest to quantum information research to investigate links between discrete variable, single photon phenomena and continuous variable, multi-photon effects. This has motivated a consideration of quantum cryptographic schemes using multi-photon light modes [4–6].

The question of optimum protocols and eavesdropper strategies has been studied in detail for the single quanta case [7], leading to general proofs of security for discrete systems. Up till now no such proofs of security have been made for continuous variable quantum cryptographic schemes. In this paper we introduce a scheme whose implementation and

evaluation is sufficiently simple that a general proof of *minimum guaranteed security* can be derived. It is shown that in principle levels of security approaching those of single quanta schemes can be achieved.

The paper is laid out in the following way. In section II we review a coherent state scheme which provides limited security but none-the-less serves to illustrate the basic concepts. In section III we derive in a general way the minimum disturbance Eve can make to the information Bob receives for a particular level of interception. This relationship is then used to make a general evaluation of the coherent scheme of section II. In section IV we extend our discussion to a scheme employing 2-mode squeezed states. We show that the 2-mode scheme can, in principle, provide high levels of guaranteed security. In section V we make the interesting observation that teleportation represents an optimum eavesdropper strategy for this system and in section VI we conclude.

II. COHERENT STATE QUANTUM CRYPTOGRAPHY

In this section we will introduce the basic idea of our continuous variable cryptographic technique by reviewing a coherent state scheme [4] which though not very secure, is illustrative. In both of our schemes we will consider encoding key information as small signals carried on the amplitude and and phase quadrature amplitudes of the beam. These are the analogues of position and momentum for a light mode and hence are continuous, conjugate variables. Although simultaneous measurements of these non-commuting observables can be made in various ways, for example splitting the beam on a 50:50 beamsplitter and then making homodyne measurements on each beam, the information that can be obtained is strictly limited by the generalized uncertainty principle for simultaneous measurements [8,9]. If an ideal measurement of one quadrature amplitude produces a result with a signal to noise of

$$(S/N)^{\pm} = \frac{V_s^{\pm}}{V_n^{\pm}} \tag{1}$$

then a simultaneous measurement of both quadratures cannot give a signal to noise result in excess of

$$(S/N)^{\pm}_{sim} = (\frac{\eta^{\pm} V_s^{\pm}}{\eta^{\pm} V_n^{\pm} + \eta^{\mp} V_m^{\pm}}) S/N^{\pm} \tag{2}$$

Here $V_s^{\pm}$ and $V_n^{\pm}$ are, respectively, the signal and noise power of the amplitude $(+)$ or phase $(-)$ quadrature at a particular rf (radio frequency) with respect to the optical carrier. The quantum noise which is inevitably added when dividing the mode is $V_m^{\pm}$. The splitting ratio is $\eta^{\pm}$ and $\eta^{+} = 1 - \eta^{-}$ (e.g a 50:50 beamsplitter has $\eta^{+} = \eta^{-} = 0.5$). The spectral powers are normalized to the quantum noise limit (QNL) such that a coherent beam has $V_n^{\pm} = 1$. Normally the partition noise will also be at this limit ($V_m^{\pm} = 1$). For a classical light field, i.e. where $V_n^{\pm} >> 1$ the penalty will be negligible. However for a coherent beam a halving of the signal to noise for both quadratures is unavoidable when the splitting ratio is a half. The Hartley-Shannon law [10] applies to Gaussian, additive noise, communication channels such as we will consider here. It shows, in general, that if information of a fixed bandwidth

is being sent down a communication channel at a rate corresponding to the channel capacity and the signal to noise is reduced, then errors will inevitably appear at the receiver. Thus, under such conditions, any attempt by an eavesdropper to make simultaneous measurements will introduce errors into the transmission. In the following we will first examine what level of security is guaranteed by this uncertainty principle if a coherent state mode is used. We will then show that the level of security can in principle be made as strong as for the single quanta case by using a special type of two-mode squeezed state.

Consider the set up depicted in Fig.1. A possible protocol is as follows. Alice generates two independent random strings of numbers and encodes one on the phase quadrature, and the other on the amplitude quadrature of a bright coherent beam. Bob uses homodyne detection to detect either the amplitude or phase quadrature of the beam when he receives it. He swaps randomly which quadrature he detects. On a public line Bob then tells Alice at which quadrature he was looking, at any particular time. They pick some subset of Bob's data to be the test and the rest to be the key. For example, they may pick the amplitude quadrature as the test signal. They would then compare results for the times that Bob was looking at the amplitude quadrature. If Bob's results agreed with what Alice sent, to within some acceptable error rate, they would consider the transmission secure. They would then use the undisclosed phase quadrature signals, sent whilst Bob was observing the phase quadrature, as their key. By randomly swapping which quadrature is key and which is test throughout the data comparison an increased error rate on either quadrature will immediately be obvious.

Let us first quantify our results for some specific situations to illustrate the essential points. Consider the specific encoding scheme of binary pulse code modulation, in which the data is encoded as a train of 1 and 0 electrical pulses which are impressed on the optical beam at some rf using electro-optic modulators. The amplitude and phase signals are imposed at the same frequency with equal power. Let us now consider some specific strategies Eve could adopt (see Fig.2). Eve could guess which quadrature Bob is going to measure and measure it herself (Fig.2(a)). She could then reproduce the digital signal of that quadrature and impress it on another coherent beam which she would send on to Bob. She would learn nothing about the other quadrature through her measurement and would have to guess her own random string of numbers to place on it. When Eve guesses the right quadrature to measure Bob and Alice will be none the wiser, however, on average 50% of the time Eve will guess wrong. Then Bob will receive a random string from Eve unrelated to the one sent by Alice. These will agree only 50% of the time. Thus Bob and Alice would see a 25% bit error rate in the test transmission if Eve was using this strategy. This is analogous to the result for single quanta schemes in which this type of strategy is the only available. Another single measurement strategy Eve could use is to do homodyne detection at a quadrature angle half-way between phase and amplitude. This fails because the signals become mixed. Thus Eve can tell when both signals are 0 or both are 1 but she cannot tell the difference between 1,0 and 0,1. This again leads to a 25% bit error rate.

However, for bright beams it is possible to make simultaneous measurements of the quadratures, with the caveat that there will be some loss of information. So a second strategy that Eve could follow would be to split the beam in half, measure both quadratures and impose the information obtained on the respective quadratures of another coherent beam which she sends to Bob (Fig.2(b)). How well will this strategy work? Suppose Alice

wishes to send the data to Bob with a bit error rate (BER) of about 1%. For bandwidth limited transmission of binary pulse code modulation [11]

$$BER = \frac{1}{2} erfc \frac{1}{2} \sqrt{\frac{1}{2} S/N} \tag{3}$$

Thus Alice must impose her data with a S/N of about 13dB. For simultaneous measurements of a coherent state the signal to noise obtained is halved (see Eq.2). As a result, using Eq.3, we find the information Eve intercepts and subsequently passes on to Bob will only have a BER of 6%. This is clearly a superior strategy and would be less easily detected. Further more Eve could adopt a third strategy of only intercepting a small amount of the beam and doing simultaneous detection on it (Fig.2(c)). For example, by intercepting 16% of the beam, Eve could gain information about both quadratures with a BER of 25% whilst Bob and Alice would observe only a small increase of their BER to 1.7%. In other words Eve could obtain about the same amount of information about the key that she could obtain using the "guessing" strategy, whilst being very difficult to detect, especially in the presence of losses.

III. OPTIMUM EAVESDROPPER STRATEGY

It is already clear from the preceding discussion that the coherent scheme does not provide good security. However our evaluation has been in terms of specific eavesdropper strategies. This is unsatisfactory for evaluating the security of more promising schemes. Instead we wish to be able to evaluate our schemes' security against attack from some theoretical, optimum eavesdropper strategy. Our approach is to identify the minimum disturbance allowed by quantum mechanics to the information Bob receives given a particular level of interception by Eve. We are then able to derive the minimum BER that Bob and Alice can find for a given BER in the information that Eve intercepts. We choose to couch our evaluation in terms of BER's because they represent an unambiguous, directly observable measure of the extent to which Eve can intercept information and the resulting corruption of Bob's information. Depending on the particular technique Eve uses Bob and Alice may be able to gain additional evidence for Eve's presence by comparing the absolute noise levels of the sent and received signals. This can only increase the security of the system. By considering a general limit on BER's we can find a minimum guaranteed security against eavesdropping regardless of the technique Eve employs.

A more general statement of the generalized uncertainty principle [9] requires that for *any* simultaneous measurements of conjugate quadrature amplitudes

$$V_M^+ V_M^- \geq 1 \tag{4}$$

where $V_M^{\pm}$ are the measurement penalties for the amplitude $(+)$ and phase $(-)$ quadratures, normalized to the amplification gain between the system observables and the measuring apparatus. For example suppose an attempt to measure the amplitude quadrature variance of a system V_k^+ returned the result $G_1 V_k^+ + G_2 V_m^+$ where V_m^+ represents noise. Then we would have $V_M^+ = (G_2/G_1) V_m^+$. Eq.2 follows directly from Eq.4 for ideal simultaneous measurements. Let us investigate what general restrictions this places on the information that Eve

can intercept and the subsequent corruption of Bob's signal. Firstly Eve's measurements will inevitably carry measurement penalties $V_E^{\pm}$ constrained by

$$V_E^+ V_E^- \geq 1 \tag{5}$$

Now suppose Bob makes an ideal (no noise added) amplitude measurement on the beam he receives. In order to satisfy Eq.4 it must be true that the noise penalty carried on the amplitude quadrature of this beam V_B^+ due to Eve's intervention, is sufficiently large such that

$$V_B^+ V_E^- \geq 1 \tag{6}$$

Similarly, Bob can also choose to make ideal measurements of the phase quadrature so we must also have

$$V_E^+ V_B^- \geq 1 \tag{7}$$

Eqs.5,6,7 are the quantum mechanical basis for our measure of minimum guaranteed security. They set strict limits on the minimum disturbance Eve can cause to Bob's information given a particular maximum quality of the information she receives. This applies regardless of the method she uses to eavesdrop. Given a particular encoding scheme, bandwidth and initial signal to noise, we can calculate minimum BER's from these results.

Let us analyze the coherent state scheme using Eqs.5,6,7. In general the signal transfer coefficients, defined as the ratio of the output to input signal to noises are given by

$$\begin{aligned} T_E^+ &= \frac{(S/N)_{eve}^+}{(S/N)_{in}^+} = \frac{V_{in}^+}{V_{in}^+ + V_E^+} \\ T_E^- &= \frac{(S/N)_{eve}^-}{(S/N)_{in}^-} = \frac{V_{in}^-}{V_{in}^- + V_E^-} \\ T_B^+ &= \frac{(S/N)_{bob}^+}{(S/N)_{in}^+} = \frac{V_{in}^+}{V_{in}^+ + V_B^+} \\ T_B^- &= \frac{(S/N)_{bob}^-}{(S/N)_{in}^-} = \frac{V_{in}^-}{V_{in}^- + V_B^-} \end{aligned} \tag{8}$$

Substituting Eqs.8 into Eqs.5,6,7 and using the fact that $V_{in}^{\pm} = 1$ we find

$$\begin{aligned} T_E^+ + T_E^- &\leq 1 \\ T_E^+ + T_B^- &\leq 1 \\ T_B^+ + T_E^- &\leq 1 \end{aligned} \tag{9}$$

Eqs.9 clearly show that any attempt by Eve to get a good signal to noise on one quadrature (e.g. $T_E^+ \rightarrow 1$) results not only in a poor signal to noise in her information of the other quadrature (e.g. $T_E^- \rightarrow 0$) but also a poor signal to noise for Bob on that quadrature (e.g. $T_B^- \rightarrow 0$), making her presence obvious. This is the general limit of the guessing strategy presented in the last section and leads to the same error rates.

Because of the symmetry of Bob's readout technique Eve's best approach is a symmetric attack on both quadratures. Eqs.9 then reduces to two equations

$$2T_E^{\pm} \leq 1$$
$$T_E^{\pm} + T_B^{\pm} \leq 1 \tag{10}$$

If Eve extracts her maximum allowable signal to noise transfer, $T_E^{\pm} = 0.5$, then ideally Bob suffers the same penalty $T_B^{\pm} = 0.5$. This is the general limit of the second strategy of the previous section. The same reduction in Bob's signal to noise occurs as in the specific implementation (Fig.2(b)) thus this implementation can be identified as an optimum eavesdropper strategy for obtaining maximum simultaneous information about both quadratures.

Eve's best strategy is to intercept only as much information as she can without being detected. The system will be secure if that level of information is negligible. Suppose, as in the last section, Eve only intercepts a signal transfer of $T_E^{\pm} = .08$. From Eq.10 this means Bob can receive at most a signal transfer of $T_B^{\pm} = .92$. This is greater than the result for the specific implementation shown in Fig.2(c), thus that implementation is not an optimum eavesdropper strategy. Using the optimum eavesdropper strategy the error rates for the specific encoding scheme discussed in the last section will be: if Eve intercepts information with a BER of 25%, then the minimum BER in Bob's information will be 1.4%.

In this section we have derived a limit to the minimum back-action that Eve can produce on Bob's signal for a given quality of her intercepted signal. This limit is completely general, not dependent on the specific eavesdropping scheme employed. We have used our limits to investigate more generally the security of the coherent state scheme, coming to the same conclusion, i.e. that it is not very secure.

IV. SQUEEZED STATE QUANTUM CRYPTOGRAPHY

The preceding discussion has shown that a cryptographic scheme based on coherent light provides much less security than single quanta schemes. We now consider whether squeezed light can offer improved security. For example amplitude squeezed beams have the property $V_n^+ < 1 < V_n^-$. Because the amplitude quadrature is sub-QNL greater degradation of S/N than the coherent case occurs in simultaneous measurements of amplitude signals (see Eq.2). Unfortunately the phase quadrature must be super-QNL, thus there is less degradation of S/N for phase signals. As a result the total security is in fact less than for a coherent beam. However in the following we will show that by using two squeezed light beams, security comparable to that achieved with single quanta can be obtained. The following scheme is similar to that presented in our previous publication [4] except that now only a single quantum limited beam is sent from Alice to Bob. This removes the possibility of Eve making a coherent attack, a loophole which (contrary to the claims in Ref. [4]) was not adequately resolved in the previous protocol.

The set-up is shown in Fig.3. Once again Alice encodes her number strings digitally, but now she impresses them on the amplitude quadratures of two, phase locked, amplitude squeezed beams, a and b, one on each. A $\pi/2$ phase shift is imposed on beam b and then they are mixed on a 50:50 beamsplitter. The resulting output modes, c and d, are given by

$$c = \sqrt{\frac{1}{2}}(a + ib)$$

$$d = \sqrt{\frac{1}{2}}(a - ib) \tag{11}$$

These beams are now in an entangled state which will exhibit Einstein, Podolsky, Rosen (EPR) type correlations [12,14]. Negligible information about the signals can be extracted from the beams individually because the large fluctuations of the anti-squeezed quadratures are now mixed with the signal carrying squeezed quadratures. One of the beams, say c, is transmitted to Bob. The other beam, d, Alice retains and uses homodyne detection to measure either its amplitude or phase fluctuations, with respect to a local oscillator in phase with the original beams a and b. She randomly swaps which quadrature she measures, and stores the results. Bob, upon receiving beam c, also randomly chooses to measure either its amplitude or phase quadrature and stores his results. After the transmission is complete Alice sends the results of her measurements on beam d to Bob on an open channel. About half the time Alice will have measured a different quadrature to Bob in a particular time window. Bob discards these results. The rest of the data corresponds to times when they both measured the same quadratures. If they both measured the amplitude quadratures of each beam Bob adds them together, in which case he can obtain the power spectrum

$$\begin{aligned} V^+ &= <|(\tilde{c}^\dagger + \tilde{c}) + (\tilde{d}^\dagger + \tilde{d})|^2> \\ &= V_{s,a} + V_{n,a}^+ \end{aligned} \tag{12}$$

where the tilde indicate Fourier transforms. Thus he obtains the data string impressed on beam a, $V_{s,a}$, imposed on the sub-QNL noise floor of beam a, $V_{n,a}^+$. Alternatively if they both measured the phase quadratures of each beam, Bob subtracts them, in which case he can obtain the power spectrum

$$\begin{aligned} V^- &= <|(\tilde{c}^\dagger - \tilde{c}) - (\tilde{d}^\dagger - \tilde{d})|^2> \\ &= V_{s,b} + V_{n,b}^+ \end{aligned} \tag{13}$$

i.e. he obtains the data string impressed on beam b, $V_{s,b}$, imposed on the sub-QNL noise floor of beam b, $V_{n,b}^+$. Thus the signals lie on conjugate quadratures but *both* have sub-QNL noise floors. This is the hallmark of the EPR correlation [15]. As for the coherent state case Alice and Bob now compare some sub-set of their shared data and check for errors. If the error rate is sufficiently low they deem their transmission secure and use the undisclosed sub-set of their data as their key.

Consider now eavesdropper strategies. Eve must intercept beam c if she is to extract any useful information about the signals from the classical channel (containing Alice's measurements of beam d) sent later. She can adopt the guessing strategy by detecting a particular quadrature of beam c and then using a similar apparatus to Alice's to re-send the beam and a corresponding classical channel later. As before she will only guess correctly what Bob will measure half the time thus introducing a BER of 25%. Instead she may try simultaneous detection of both quadratures of beam c. As in the coherent case the noise she introduces into her own measurement ($V_E^\pm$) and that she introduces into Bob's ($V_B^\pm$) are in general limited according to Eqs.5,6 and 7. However now the consequences of these noise limits on the signal to noise transfers that Eve and Bob can obtain behave quite differently because the signals they are trying to extract lie on sub-QNL backgrounds. Eve's signal transfer coefficients are given by

$$T_E^+ = \frac{V_{n,a}^+}{V_{n,a}^+ + 0.5V_E^+}$$
$$T_E^- = \frac{V_{n,b}^+}{V_{n,b}^+ + 0.5V_E^-} \tag{14}$$

and similarly Bob's are

$$T_B^+ = \frac{V_{n,a}^+}{V_{n,a}^+ + 0.5V_B^+}$$
$$T_B^- = \frac{V_{n,b}^+}{V_{n,b}^+ + 0.5V_B^-} \tag{15}$$

For the squeezed noise floors the same ($V_{n,a}^+ = V_{n,b}^+ = V_n$) we find the signal transfers are restricted via

$$4V_n^2(\frac{1}{T_E^+} - 1)(\frac{1}{T_E^-} - 1) \geq 1 \tag{16}$$

$$4V_n^2(\frac{1}{T_E^+} - 1)(\frac{1}{T_B^-} - 1) \geq 1 \tag{17}$$

$$4V_n^2(\frac{1}{T_B^+} - 1)(\frac{1}{T_E^-} - 1) \geq 1 \tag{18}$$

It is straightforward to show that a symmetric attack on both quadratures is Eve's best strategy as it leads to a minimum disturbance in both her and Bob's measurements. Using this symmetry to simplify Eq.16 leads to the following general restriction on the signal transfer Eve can obtain:

$$T_E^{\pm} \leq \frac{2V_n}{2V_n + 1} \tag{19}$$

Once the squeezing exceeds 3 dB ($V_n = 0.5$) the signal to noise that Eve can obtain simultaneously is reduced below that for the coherent state scheme. In the limit of very strong squeezing ($V_n \rightarrow 0$) Eve can extract virtually no information simultaneously. Similarly Bob's signal transfer is restricted according to:

$$\frac{T_E^{\pm} T_B^{\pm}}{(1 - T_E^{\pm})(1 - T_B^{\pm})} \leq 4V_n \tag{20}$$

If squeezing is strong then almost any level of interception by Eve will result in very poor signal transfer to Bob. As a numerical example consider the specific encoding scheme of section 2 and suppose the squeezing is 13 dB ($V_n = 0.05$). If Eve makes an ideal simultaneous measurement then both she and Bob will obtain a signal transfer of .09. As a result,assuming initial S/N of 13dB and using Eqs3, 19, 20 we find the information Eve intercepts and Bob receives will both have a minimum BER of about 24%. In other words, the security against an eavesdropper using simultaneous measurements is now on a par with the guessing strategy.

Eve must reduce the amount of information she intercepts to virtually nothing before Bob's change in BER becomes negligible. The trade-off between Eve and Bob's BER's are shown graphically in Fig.4 for various levels of squeezing. Also shown is the corresponding trade-off for an ideal single quanta scheme. For high levels of squeezing the results are comparable.

In any realistic situation losses will be present. Losses tend in general to reduce security in quantum cryptographic schemes [16]. Eve can take advantage of losses by setting up very close to Alice and effectively disguising herself as loss. In such a situation her signal transfer remains as in Eq.19 but Bob's changes to

$$(\frac{1}{T_E^{\pm}} - 1)(\frac{1}{T_B^{\pm}} - 1 - \frac{(1-\eta)}{2\eta V_n}) \geq \frac{1}{4V_n} \tag{21}$$

where η is the transmission efficiency. In Fig.5 some numerical examples including loss are given. Although these examples demonstrate some tolerance to loss for our continuous variable system it should be noted that single quanta schemes can tolerate much higher losses [17] making them more practical from this point of view.

V. TELEPORTATION AS THE OPTIMUM EAVESDROPPER STRATEGY

In this section we show that Eve can use continuous variable teleportation [18,19,14] as an optimum eavesdropper strategy. Quantum teleportation uses shared entanglement to convert quantum information into classical information and then back again (see Fig.6). In particular continuous variable teleportation uses 2-mode squeezed light as its entanglement resource. In the limit of very strong squeezing no information about the teleported system can be extracted from the classical channel but a perfect reproduction of the quantum system can be retrieved. On the other hand with lower levels of squeezing some information about the system can be obtained from the the classical channel but at the expense of a less than perfect reproduction. We show in the following that under particular operating conditions the disturbance in the teleported state is precisely the minimum required by the generalized uncertainty principle, given the quality of information that can be extracted from the classical channel. Teleportation thus constitutes an optimum eavesdropper strategy.

Eves strategy would be to send the field she intercepts from Alice through a teleporter, adjusted such that she can read some information out of the classical channel, but still reconstruct the field sufficiently well such that Bob and Alice don't see a large BER. The classical channel of a lossless continuous variable teleporter can be written [4,14]

$$\begin{aligned} F_c &= K(f_{in} + j_1^{\dagger}) \\ &= K(f_{in} + \sqrt{G} v_1^{\dagger} + \sqrt{G-1} v_2) \end{aligned} \tag{22}$$

where f_{in} is the annihilation operator of the input to the teleporter and $j_1 = \sqrt{G} v_1 + \sqrt{G-1} v_2^{\dagger}$ is the annihilation operator for one of the entangled beams. The v_i are the vacuum mode inputs to the squeezers, G is the parametric gain of the squeezers and $K >> 1$ is the measurement amplification factor. Being a classical channel simultaneous measurements of both quadratures can be made without additional penalty thus immediately Eve's measurement penalty is

$$V_E^{\pm} = 2G - 1 \tag{23}$$

For no squeezing ($G = 1$) $V_E^{\pm} = 1$, the minimum possible for simultaneous detection of both quadratures (see Eq.5). For large squeezing ($G >> 1$) $V_E^{\pm}$ become very large and Eve can obtain little information from the classical channel.

The output of the teleporter is given by

$$\begin{aligned} f_{out} &= \lambda f_{in} + j_1^{\dagger} - j_2 \\ &= \lambda f_{in} + (\lambda\sqrt{G} - \sqrt{G-1})v_1^{\dagger} + (\sqrt{G} - \lambda\sqrt{G-1})v_2 \end{aligned} \tag{24}$$

where λ is the gain of the teleporter and $j_2 = \sqrt{G}v_2 + \sqrt{G-1}v_1^{\dagger}$ is the annihilation operator for the other entangled beam. Thus Bob's measurement penalty for ideal measurements of either of the quadratures is

$$V_B^{\pm} = \frac{(\lambda\sqrt{G} - \sqrt{G-1})^2 + (\sqrt{G} - \lambda\sqrt{G-1})^2}{\lambda^2} \tag{25}$$

If Eve operates the teleporter with gain [20]

$$\lambda_{opt} = \frac{1 + V_{sq}^2}{1 - V_{sq}^2} \tag{26}$$

where $V_{sq} = (\sqrt{G} - \sqrt{G-1})^2$, then Bob's noise penalty is

$$V_B^{\pm}(\lambda_{opt}) = \frac{1}{2G-1} \tag{27}$$

and so Eve causes the minimum allowable disturbance, i.e. $V_E^{\pm}V_B^{\pm} = 1$.

VI. CONCLUSION

We have derived quantum mechanical limits to the minimum disturbance that can be observed in the data shared by Alice and Bob in a continuous variable cryptographic scheme, in the presence of an eavesdropper using an optimum eavesdropper strategy. Using these limits we have shown, for the first time, a continuous variable scheme which can, in principle, have a guaranteed minimum security comparable with those of discrete, single quanta systems. In practice the high levels of squeezing and low levels of loss required will restrict the applicability of this particular scheme. Never-the-less we believe this work can be viewed as a simple, but rigorous starting point from which more practical schemes may be developed. Some promising lines for future inquiry are: investigating different encoding and transmission protocols; developing tighter security bounds and; investigating the use of higher order entanglement.

We have shown that continuous variable teleportation is an optimum eavesdropper strategy for our system. This identifies continuous variable teleportation as the optimum method for extracting information from a quantum object whilst causing the least possible disturbance.

Most generally this system is an example of a new quantum information technology based on continuous variable, multi-photon manipulations. Such technologies may herald a new approach to quantum information.

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FIGURES

FIG. 1. Schematic of coherent light cryptographic set-up. AM is an amplitude modulator whilst PM is a phase modulator.

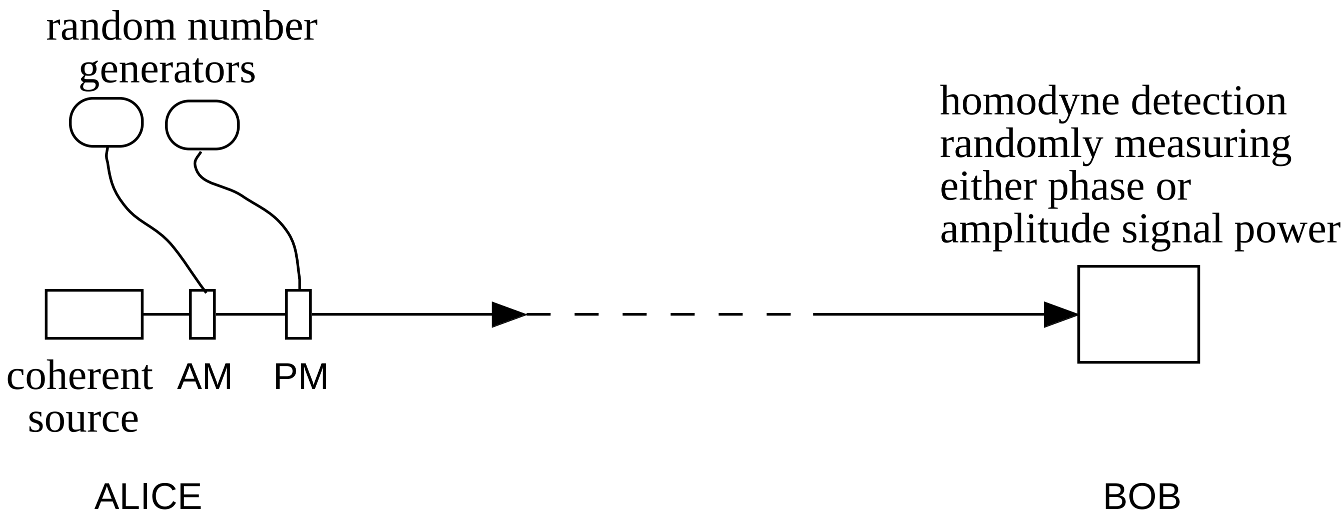
FIG. 2. Schemata of three eavesdropper strategies. Only (a) is available in single quanta schemes.

FIG. 3. Schematic of squeezed light cryptographic set-up. Sqza and sqzb are phase locked squeezed light sources. Rna and Rnb are independent random number sources. Bs and pbs are non-polarizing and polarizing beamsplitters respectively. Half-wave plates to rotate the polarizations are indicated by $\lambda/2$ and optical amplification by A. The $\pi/2$ phase shift is also indicated. HD stands for homodyne detection system.

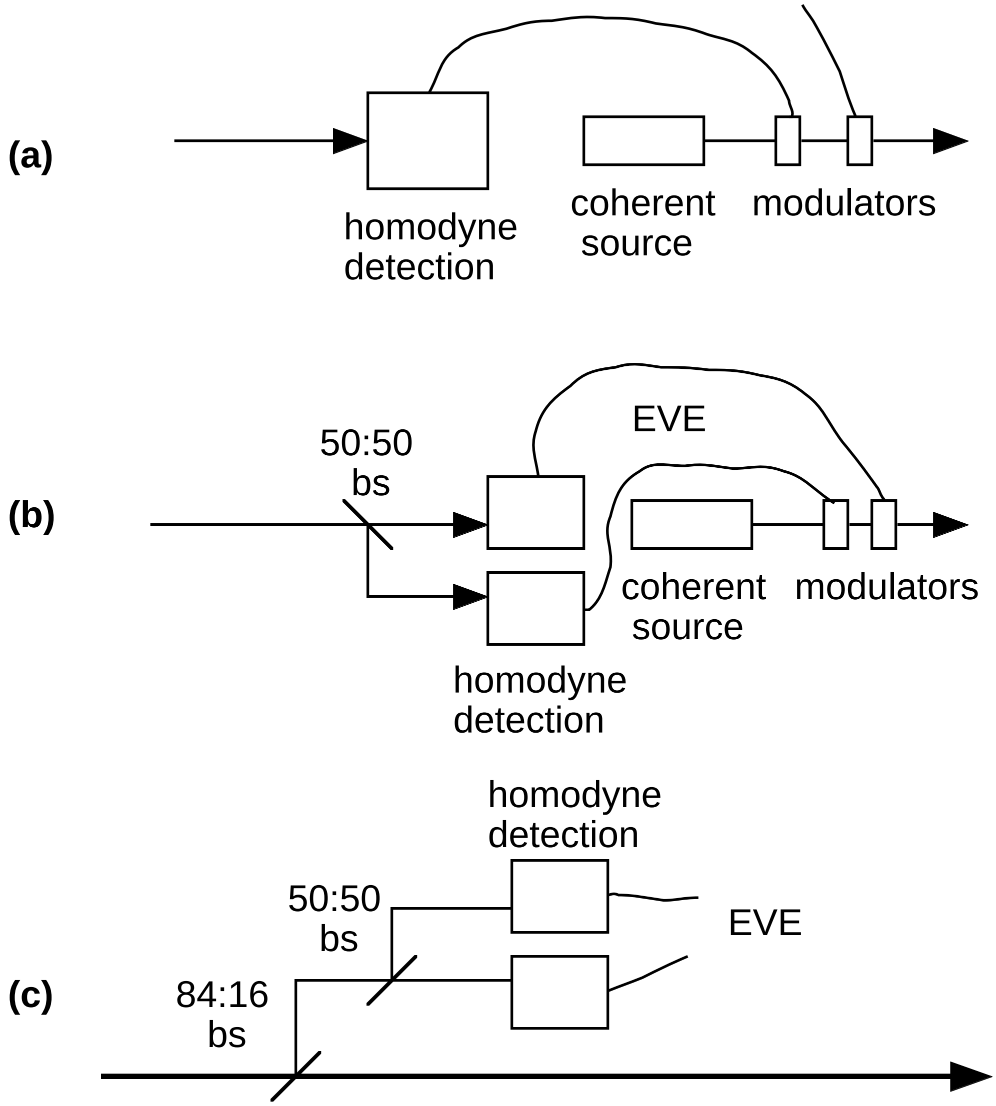
FIG. 4. Minimum guaranteed security of squeezed light scheme. Minimum allowable BER's in the data of Bob and Eve are plotted for various levels of squeezing. The dotted line is for an ideal single quanta system.

FIG. 5. Minimum guaranteed security of squeezed light scheme. Minimum allowable BER's in the data of Bob and Eve are plotted for various levels of transmission efficiency for 95% squeezing.

FIG. 6. Schematic of teleportation being used as an optimum eavesdropper strategy.



T.C.Ralph, "Continuous Variable...", Fig. 1



T.C.Ralph, "Continuous Variable...", Fig. 2

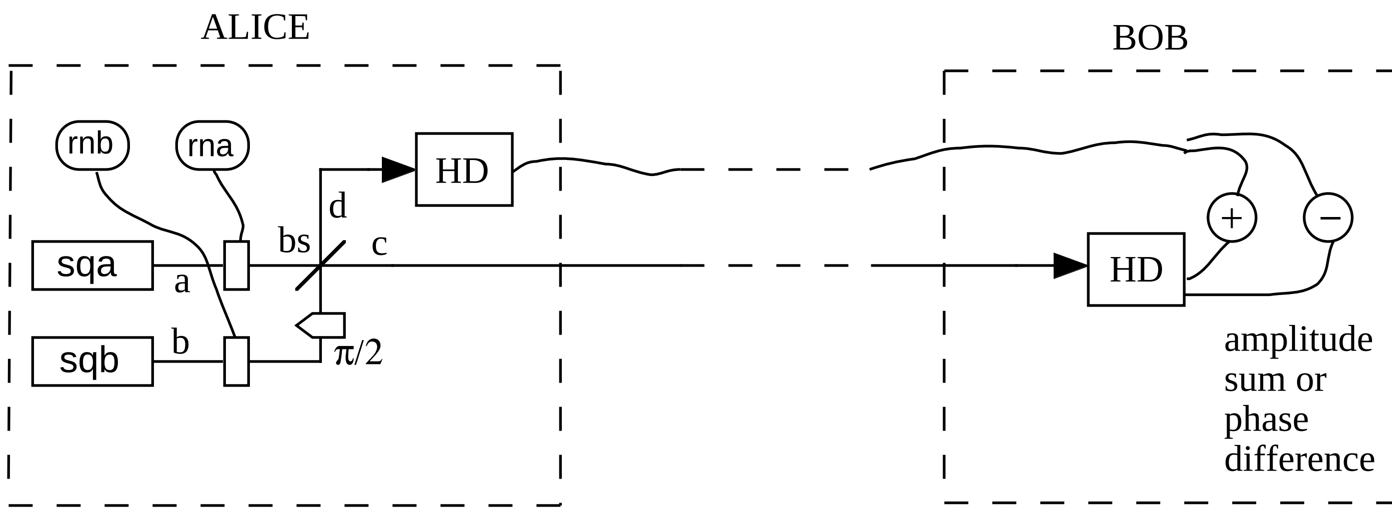


Figure 3
"Security of Continuous ...", Ralph

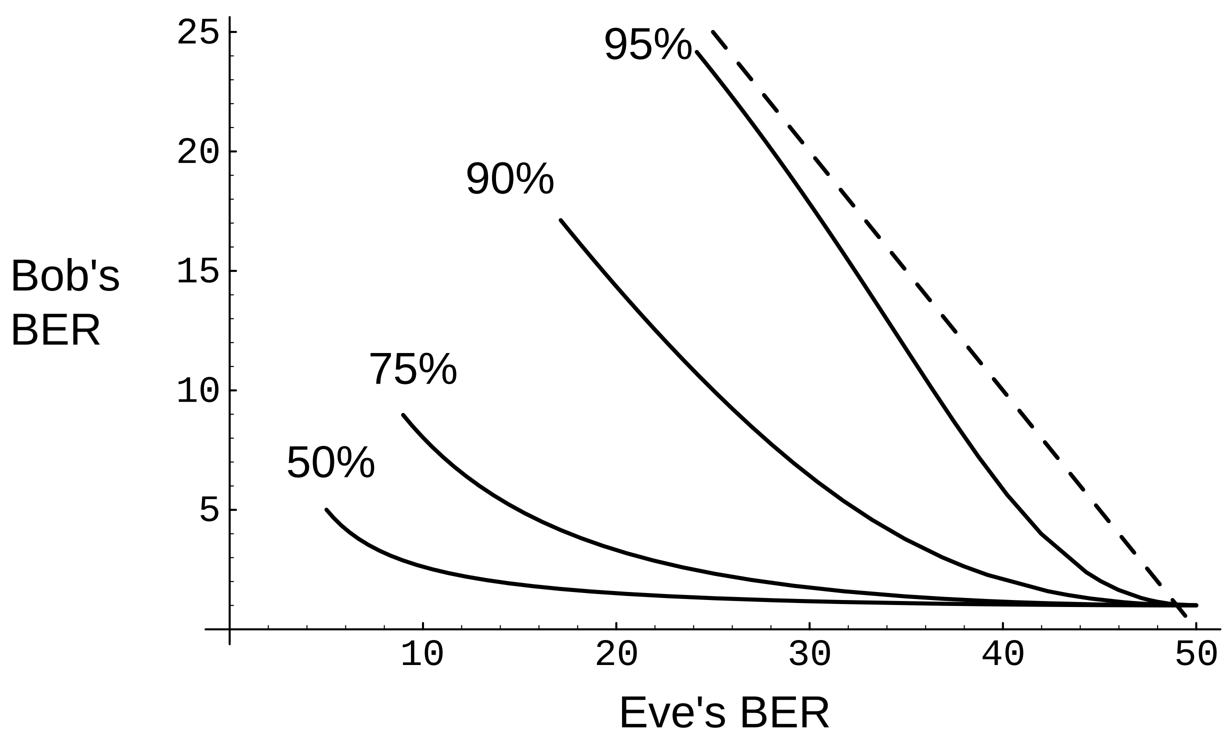


Figure 4
"Security of Continuous..." Ralph

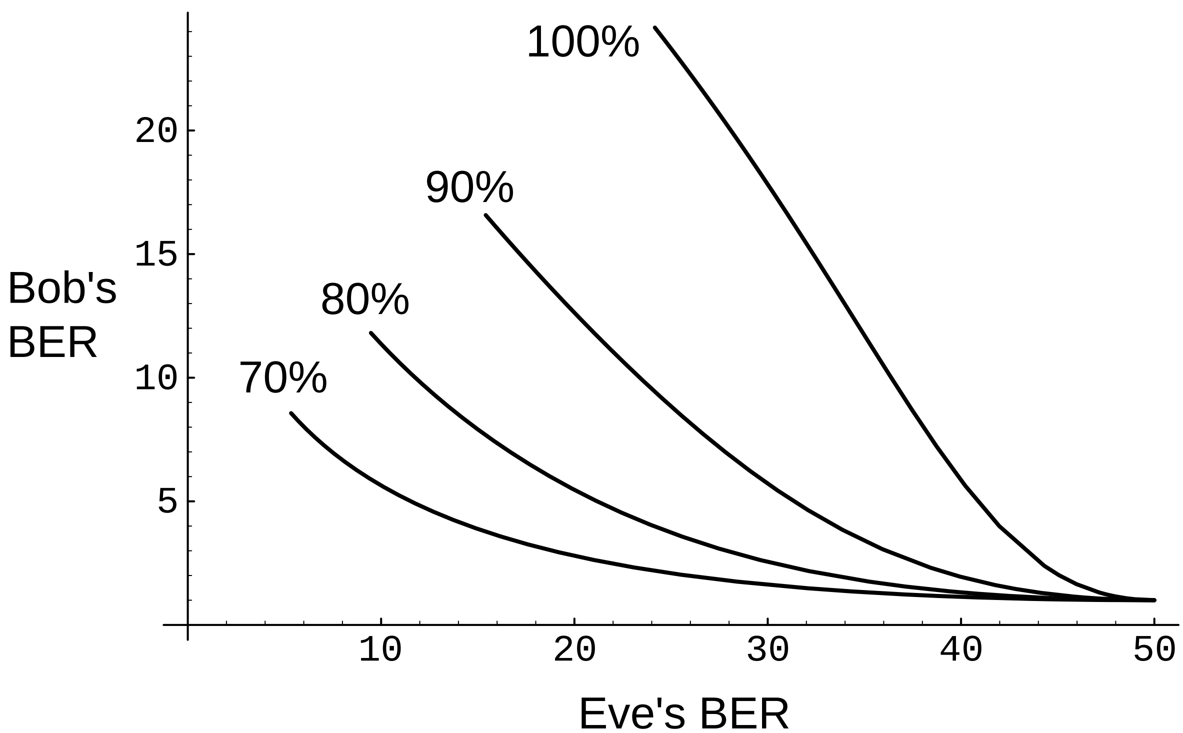
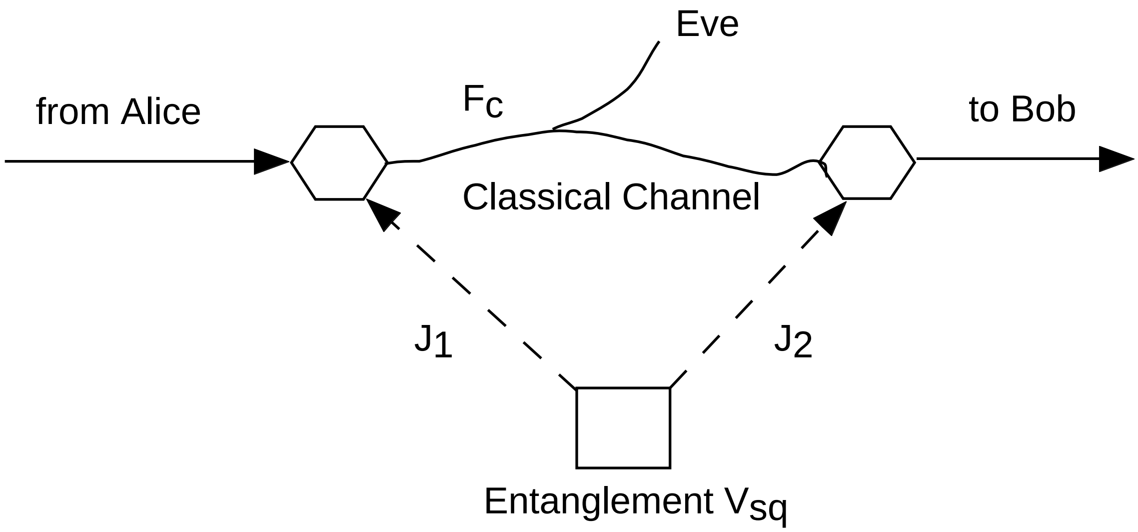


Figure 5
"Security of Continuous..." Ralph


Eve
from Alice
Fc
to Bob
Classical Channel
J1
J2
Entanglement Vsq